Influence of Nuclear Spin Polarization on Quantum Wire Conductance

James A. Nesteroff, Yuriy V. Pershin and Vladimir Privman

***Abstract*— In this work, we study a possibility to measure the transverse and longitudinal relaxation times of a collection of polarized nuclear spins located in the region of a quantum wire via its conductance. The interplay of an external in-plane magnetic field, spin-orbit interaction, and the changing field of the spin-polarized nuclei cause the conductance of the quantum wire to evolve in time. We show that it is possible to extract the transverse and longitudinal relaxation times of the spin-polarized nuclei from the time dependence of the conductance.**

I. Introduction

In the present work we investigate the conductance of structures, which fall into the realm of mesoscopic. This regime is characterized by length scales which lie in between the microscopic and the more familiar macroscopic world. Over the past decade and a half, there has been considerable interest in these devices by both experimentalists and theoreticians alike [1], [2]. Mesoscopic devices have also found a place in the field of spintronics, which studies novel electronic devices based on the use of electron spins [3], [4].

The quantum wire (QW) is by far one of the simplest examples of a mesoscopic device. The main principle is that it confines electron transport to one dimension only. This can be accomplished in a heterostructure, such as a AlGaAs/GaAs, which is used to confine the electrons into a two dimensional electron gas (2DEG). The application of split metallic gates on top of the 2DEG gives rise to lateral confinement. Thus, the QW is formed in between two reservoirs of electrons (or contacts). By applying a negative bias to the gates, the electrons underneath are depleted, thus forming a constriction in the 2DEG. When the width of this QW (constriction) becomes short compared to the electron Fermi wavelength, a small number of quantized modes appear. By changing the strength of the bias the width of the QW, and ultimately the number of modes in the channel, can be controlled. If the length of the quantum wire is sufficiently short, the electrons propagate through the device without suffering collisions due to impurities, phonons, or other electrons [5]. This regime of transport is referred to as *ballistic* [1], [2], [5].

Some of the most interesting properties of QW appear in this ballistic regime. For example, consider the situation when a bias is applied across the contacts. Electrons are then allowed to flow from one reservoir to the other through the QW. Since the electrons do not suffer any scattering in the conductor one might assume that there is zero resistance. Yet, this conclusion is false, in fact there is a resistance which is associated with the contacts themselves. Moreover, this resistance can be expressed in terms of fundamental constants,

$$R = \frac{h}{2e^2 M}, \tag{1}$$

where M is the number of discrete modes in the channel [5]–[7]. For a single-mode wire this resistance is approximately $12.9\text{k}\Omega$. Thus, in the limit of a large number of modes in the channel this contact resistance becomes negligible. However, if one were to vary the width of the QW in such a way as to increase the number of modes one by one, then the resistance (conductance) should decrease (increase) in a step-like fashion.

The quantization of the conductance (or conductance plateaus) in QWs was discovered in 1988 [6], [7]. Since then, there has been a large number of studies, both theoretical and experimental, of the properties of these devices. In recent years, researchers have also started to investigate the role which spin-orbit (SO) interaction, which affects the electron spin, could play in the conductance of these systems [8]–[13]. To the best of our knowledge, the effect of SO interaction on QW conductance was studied experimentally only in [14]. However, the results reported in [14] do not allow to clearly resolve the role of SO interaction in QW transport.

The SO interaction also plays a large part in the operation of many recently proposed spintronic devices, reviewed, e.g., in [4]. One of the most popular and widely cited proposals is the Datta-Das spin transistor [3]. Its design is similar to the well known Field Effect Transistor (FET), that is two contacts are connected by a channel (2DEG) with a gate controlling the concentration of electrons in the channel. There are, however, several important differences. The contacts are made from magnetic materials The source contact causes electrons entering the channel to become spin polarized. Secondly, the gate controls the strength of the Rashba spin-orbit interaction [15], [16], as opposed to a regular FET in which the gate only modulates the electron concentration. When an electron enters the channel, the spin is rotated from its original orientation by an amount proportional to the strength of the Rashba coupling [3] and the length of the channel. This phase shift can be, in principle, detected as a change in the drain-source current, since the contact at the other end of the channel acts as a spin filter [3].

It has been shown that SO interaction can modify the energy spectrum of the electrons in the channel of a QW and, conse-

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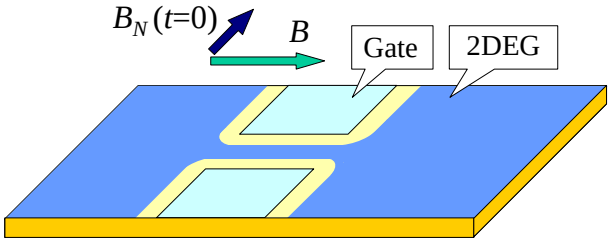

Fig. 1. Quantum wire with an applied magnetic field (of magnitude B) in the x direction, and an effective nuclear hyperfine field (of magnitude B_N) initially pointing in the y direction.

quently, affect the conductance [8]–[10]. These studies were done under the assumption of perfect transmission through the wire. Other researchers [17]–[20] have focused on wires which have transmission coefficients less than unity. In a recent paper [12], we have shown that if an in-plane magnetic field is applied to a QW in which the Rashba SO interaction is present then the energy subbands inside the channel develop local extremal points which are dependent on the magnetic field direction.

Extending this idea further [13], we have studied the influence of a collection of spin-polarized nuclei in the region of a QW on the conductance at low applied magnetic fields. A possible realization of such a system is shown in Fig. 1. The nuclear spins are initially polarized in the direction perpendicular to the QW and in the plane of the 2DEG. If a magnetic field is applied along the QW, then the nuclear spins will begin to precess around the magnetic field direction. The influence of the polarized nuclear spins on the electron spins can be described by an effective magnetic field, whose components will change in time as the nuclear spins are reoriented. It was shown that at low magnetic fields the precessing nuclear spin polarization causes the extrema of the energy bands to change in time. As a consequence, the conductance of the wire, at fixed bias, shows an interesting time variation [13]. The aim of this paper is to find the conductance in the limit of high magnetic fields and long times as well as to summarize and extend previously obtained results. We derive an expression for time dependence of the conductance that can be used for extraction of the transverse and longitudinal nuclear spin relaxation times from experimental results.

II. Overview of Physical Mechanisms

In this section will give a brief overview of the theory of transport in QW. This will be based on the well known *Landauer-Büttiker* formalism. We will also include an introduction to spin-orbit interaction concentrating on solid-state systems, and briefly discuss nuclear spin polarization in semiconductors.

A. The Landauer-Büttiker formalism

The transport properties of mesoscopic systems have been of great interest over the years [21]. Landauer was one of the first to relate the conductance of a disordered one-dimensional system to the probability that an electron can be transmitted through the system [22], [23]. Büttiker *et al.* [24] generalized this idea to account for devices which have an arbitrary number of leads and conducting channels in those leads.

The electron energy spectrum in the wire can be written as $E_n(k) = E_n^{tr} + p^2/(2m^*)$, where E_n^{tr} is the spectrum of transverse subbands, p is the momentum operator and m^* is the electron effective mass. Consider contribution to the current from the electrons of a single subband. Starting from the relation for current, $I = e\tilde{n}v$, where e is the charge of the electron, $\tilde{n}$ is the electron concentration, and v is the velocity of electrons in the channel, one can get the following expression for the current through the QW,

$$I_n = \frac{2e}{h} \int_{E_n^{tr}}^{\infty} [f(E,\mu_1) - f(E,\mu_2)]\, dE. \tag{2}$$

Here, $f(E,\mu_{1,2})$ is the Fermi distribution function evaluated at the chemical potentials $\mu_{1,2}$ of the two reservoirs that the QW connects. The prefactor $2e/h$ in (2) is a result of the cancelation of the electron velocity and the one-dimensional density of states. Using the relation, $G = I/V$, where eV, is the difference in the chemical potentials of the two reservoirs, and taking into account the contributions from all subbands, we can write the conductance as [5]

$$G = \frac{2e^2}{h} \sum_{n=0} f(E_n^{tr},\mu). \tag{3}$$

In the zero temperature limit, (3) becomes

$$G = \frac{2e^2}{h} M, \tag{4}$$

where, M is the number of modes in the channel (number of E_n^{tr} that is less than μ).

The formulas (3) and (4) were derived under the assumption that each subband has a single extremal point and no spin dependence of the band structure. A more generalized form of the Landauer-Büttiker formula for the conductance, which takes into account energy bands with an arbitrary number of local extremal points and spin dependence, is

$$G = \frac{e^2}{h} \sum_{n,s} \sum_{i} \beta_i^{n,s} f(E_i^{n,s},\mu), \tag{5}$$

where the first sum is over each spin direction (s) and each subband (n), the second sum is over subband extremal points, $E_i^{n,s}$ denotes the extremal energy, and $\beta_i^{n,s}$ is $+1$ for minima and -1 for maxima [12].

B. Spin-Orbit Interaction

The spin-orbit interaction is a contribution to the electron Hamiltonian which describes the coupling of the spin and the motional degrees of freedom [25]. The standard form assumed for the SO term in the Hamiltonian for a particle moving in an external electric field is [26]

$$H_{\mathrm{SO}} = \frac{\hbar}{4m^2c^2} (\nabla V \times \mathbf{p}) \cdot \sigma. \tag{6}$$

Here $\mathbf{p}$ is the momentum operator, m is the mass, V is the electric field potential energy, and σ represents the vector of the Pauli spin matrices.

When working with crystal structures, there are two main sources of SO coupling. The Dresselhaus SO interaction [27] is present in semiconductors lacking bulk inversion symmetry. When restricted to a two-dimensional semiconductor nanostructure with specific growth geometry ((100) growth direction [28]), this coupling is of the form

$$H_D = \beta(p_x\sigma_x - p_y\sigma_y), \tag{7}$$

where β is a constant and $p_{x,y}$ are the momentum operators in the *x* and *y* directions. Assuming that the confining potential in the region of the 2DEG channel is linear, $V(x) = Cx$ where C is a constant, the Rashba contribution to the SO interaction [15], [16] is obtained, in the form

$$H_R = \alpha(p_y\sigma_x - p_x\sigma_y) \tag{8}$$

where α is known as the Rashba constant. Both the Dresselhaus and Rashba SO couplings are responsible for lifting the two fold spin degeneracy of the electrons. We limit our consideration to the systems with only the Rashba interaction, because even in zinc-blende semiconductors it is possible to suppress the Dresselhaus coupling by the appropriate heterostructure growth protocols [29]. Generally, incorporation of the Dresselhaus interaction into our calculations is straightforward.

C. Nuclear Spin Polarization

Elemental isotopes with non-zero nuclear spin are present in all semiconductor materials. Although the nuclear spins are usually disregarded in studies of spin-polarized transport [30], it is known that in some cases nuclear spin polarization could have a significant effect on system properties. When nuclear spins in a semiconductor are polarized, the electron spins feel an effective hyperfine field, which lifts the spin degeneracy. For example, for naturally abundant isotopes in GaAs this field reaches $B_N = 5.3\,\mathrm{T}$ in the limit when all nuclear spins are completely polarized [31]. The evolution of the nuclear spin polarization can be described phenomenologically by the Bloch equations [32],

$$\frac{d\mathbf{B}_N^i}{dt} = \gamma_i \mathbf{B}_N^i \times \mathbf{B} - \frac{B_{N,y}^i\hat{y} + B_{N,z}^i\hat{z}}{T_2^i} - \frac{B_{N,x}^i - B_0^i}{T_1^i}\hat{x}, \tag{9}$$

where the index $i = 1,\ldots,p$ denotes different types of nuclear spins, γ_i denote the gyromagnetic ratios, $\mathbf{B}$ is the external magnetic field, B_0^i gives the equilibrium values for the effective magnetic fields of the nuclear spins, and $T_{1,2}^i$ are the longitudinal and transverse spin relaxation times, respectively. The total magnetic field due to the polarized nuclear spins is defined as $\mathbf{B}_N = \sum_{i=1}^p \mathbf{B}_N^i$. The equilibrium (thermal) value of the effective magnetic fields B_0^i is rather small. Generally, the following condition is fulfilled: $T_1 \gg T_2$ [32].

Assuming that only the nuclear spin isotope with $i = 1$ has non-zero polarization component in the y-direction at $t = 0$, we can easily solve the Bloch equations (9) to obtain the time dependence of the effective magnetic field of the spin-polarized nuclei,

$$B_{N,x}(t) = \sum_{i=1}^{p}(B_{N,x}^i(t=0) - B_0^i)e^{-t/T_1^i} + B_0^i, \tag{10}$$

$$B_{N,y}(t) = B_{N,y}^1(t=0)e^{-t/T_2^1}\cos(\gamma_1 Bt), \tag{11}$$

$$B_{N,z}(t) = -B_{N,y}^1(t=0)e^{-t/T_2^1}\sin(\gamma_1 Bt). \tag{12}$$

Here $B_{N,y}^1(t=0)$ and $B_{N,x}^{1,\ldots,p}(t=0)$ are the initial values of the effective magnetic fields. In what follows we will denote $\gamma \equiv \gamma_1$. Experimentally, the manipulation of nuclear spins can be accomplished by many different techniques including optical pumping [31], [33] and nuclear spin polarization by spin-polarized current [34], [35].

III. Effect of a Magnetic Fields and Spin-Orbit Interaction on Quantized Conductance

Let us consider the energy spectrum of a QW in which, due to an asymmetric confinement potential perpendicular to the 2DEG, an electron experiences Rashba SO interaction [12]. Moreover, it is assumed that QW is subjected to an in-plane magnetic field $\mathbf{B} = B_x\hat{x} + B_y\hat{y}$. We align our coordinate system such that $\hat{x}$ points along the direction of the electron transport and $\hat{y}$ is in the plane of the 2DEG and transverse to the conductor; see Fig. 1. Thus the one-particle effective-mass Hamiltonian can be written as,

$$H = \frac{\hat{p}^2}{2m^*} + V(y) - \imath\alpha\frac{\partial}{\partial x}\sigma_y + \Gamma\sigma\cdot\mathbf{B}, \tag{13}$$

where, $\Gamma = g^*\mu_B/2$, g^* is the effective electron *g* factor, and μ_B is the Bohr Magneton. It should be emphasized that the in-plane magnetic field does not enter into (13) via the vector potential in this approximation [12].

The electron energy spectrum can be found by solving the Schrödinger equation,

$$E = \frac{\hbar^2k^2}{2m^*} + E_n^{tr} \pm \Gamma\sqrt{B^2 + \frac{2\alpha kB_y}{\Gamma} + \left(\frac{\alpha k}{\Gamma}\right)^2}. \tag{14}$$

Here, B is the magnitude of the magnetic field, $B_y = B\sin(\theta)$, θ is the angle of $\mathbf{B}$ measured from the *x*-axis and E_n^{tr} are the eigenvalues of the transverse modes. Assuming the parabolic confinement potential in the y direction, $V(y)$, we have

$$E_n^{tr} = \hbar\omega(n + 1/2). \tag{15}$$

In Fig. 2, we show the energy spectrum for $\theta = \pi/4$.

It is interesting to analyze some of the features of the dispersion relation (14) and its dependence on the direction of the magnetic field. Let us start from the limiting case when the magnetic field is aligned with the *x*-axis. In this case E in (14) depends only on k^2, which results in a symmetric band structure. If the magnetic field is sufficiently small, then the lower band exhibits three local extrema (one maximum and two minima). It is easy to note that the energy splitting at $k = 0$ is equal to the Zeeman splitting energy. As the magnitude of the momentum of the electron increases, the effective Rashba field becomes stronger and causes the spin quantization axis to reorient itself along some linear combination of the magnetic

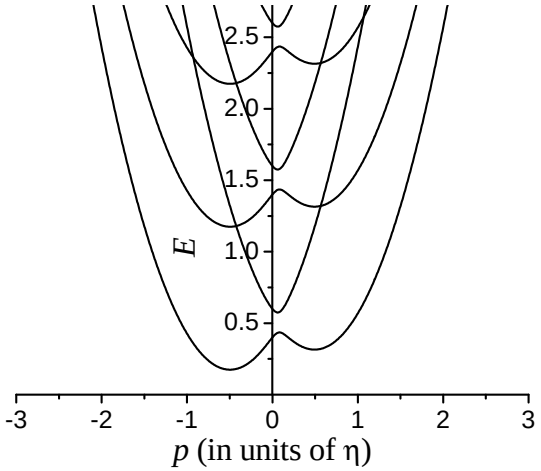

Fig. 2. Dispersion relation corresponding to $\theta = \pi/4$, with energy defined in units of $\hbar\omega$, as a function of the dimensionless momentum, p/η, where $\eta = (2m^*\hbar\omega)^{1/2}$. The parameters used in these plots were, $g^*\mu_B B/2\hbar\omega = 0.1$ and $\alpha[2m^*/(\hbar^3\omega)]^{1/2} = 1$.

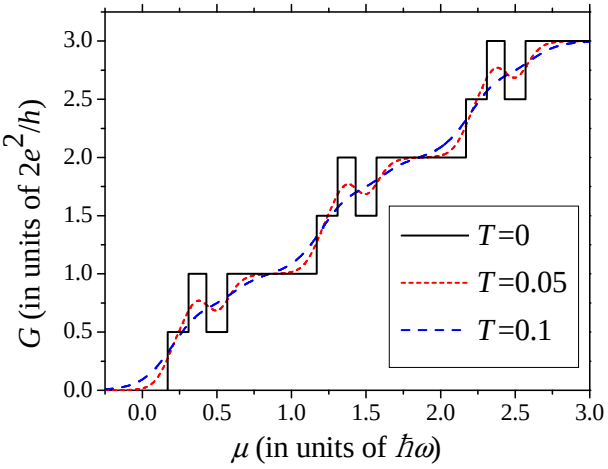

Fig. 3. Conductance as a function of the chemical potential, μ, for different temperatures T (in units of $\hbar\omega/k_B$), for $\theta = \pi/4$.

and effective Rashba field directions. In the limit of $k \to \infty$ the spin quantization axis is completely aligned with the Rashba field direction, which is along the *y*-axis. It is important to keep in mind that non-zero θ results in the asymmetry of the dispersion relation (Fig. 2). Maximal asymmetry is observed for $\theta = \pi/2$ [12].

The conductance for $\theta = \pi/4$ is shown in Fig. 3. The features of the conduction bands are reflected in the conductance, which differs drastically from the conductance in which these effects are absent (see [6], [7], [36]). If we consider the zero-temperature limit, the rise of the chemical potential past each of the subband minima results in the rise of conductance by e^2/h. When the chemical potential passes a maximum, the conductance drops by e^2/h, as given by (5). The effect of non-zero temperature is a smearing of the sharp steps of conductance observed at $T = 0$.

IV. Effect of Nuclear Spin Polarization on Conductance

Having demonstrated that the conductance of a QW is significantly effected by the presence of in-plane field and Rashba SO interaction we now turn to an application of this finding. The main goal of this section is to investigate how the nuclear spin polarization influences the QW conductance, with particular emphasis on nuclear spin relaxation time extraction from time-dependence of the conductance. In what follows we assume that the external magnetic field is applied in the x direction, along the wire, that is also the direction of the initial nuclear spin polarization. After the nuclear spins were polarized, the nuclear spins of a selected isotope are rotated in (x, y) plane from x direction, via, for example, an NMR pulse [32] at $t = 0$. Influence of such non-equilibrium nuclear spin polarization on quantum wire conductance is studied below.

In order to simplify our calculations, we assume that the evolution of the nuclear spin polarization described by (10)-(12) occurs on a time scale that is much longer than the time in which an electron traverses the QW. Thus, the energy dispersion relations could be calculated utilizing an adiabatic approximation. Then, the Hamiltonian of the system and the electron energy spectrum are given by (13) and (14) respectively with the external magnetic field $\mathbf{B}$ replaced by $\mathbf{B}_t = \mathbf{B} + \mathbf{B}_N$.

Below, we calculate the conductance of QW in two cases. We show that in low magnetic fields, when the energy of the Zeeman splitting is comparable with the relevant spin-orbit coupling energy, the conductance of this system exhibits damped oscillations on the time scale T_2 and a smooth evolution on the time scale T_1. In the second case, of high magnetic fields, the effect of the spin-orbit interaction can be neglected. Within this approximation, the conductance evolution is smooth on both time scales. We note that in order to observe the effect of the nuclear spin polarization on the conductance, the gate voltage should be properly selected: extremal subband points should be close to the chemical potential of the QW.

A. Low Magnetic Fields

In this subsection we consider the effect of nuclear spin polarization on QW conductance at short times (on the time scale of T_2), characterized by non-zero transverse component of the nuclear spin polarization, and at low magnetic fields. We have already demonstrated in [13] that the interplay of SO interaction with precessing nuclear spin polarization results in time-dependent electron energy spectrum and QW conductance. Fig. 4 shows the electron energy spectrum at different moments of time. One can see that the time dependence of the nuclear hyperfine field causes the energy spectrum given by (14) to have oscillating extremal points in time, as demonstrated in Fig. 4. Only the local extrema contribute to the conductance; see (5). Thus, the time dependent conductance of this system will exhibit damped oscillations, as shown in Fig. 5. It is interesting to note that there are two conductance oscillations per one period of nuclear spin precession around the external magnetic field.

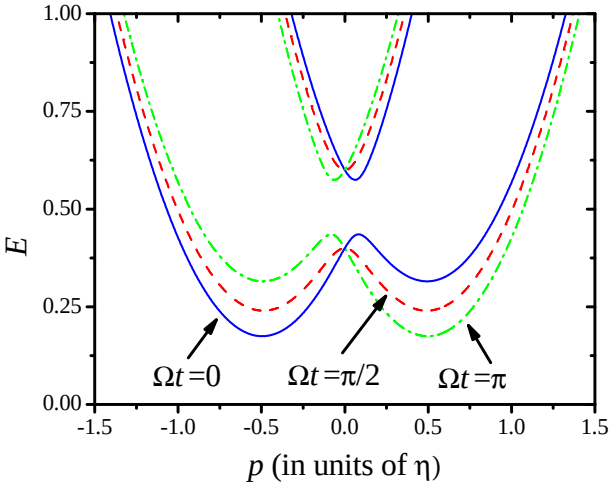

Fig. 4. The lowest-energy sub-bands, in units of $\hbar\omega$, as functions of $p = \hbar k$, for three different times. Here $\eta = (2m^*\hbar\omega)^{1/2}$ and $\Omega = \gamma B$. The two sets of curves correspond to spin $\pm$.

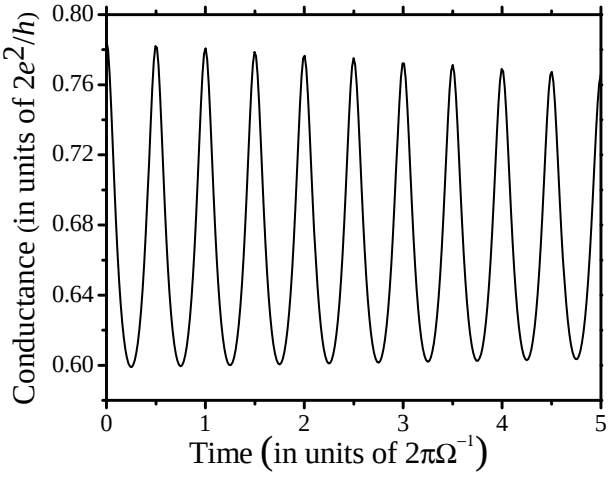

Fig. 5. Time dependence of the conductance at a finite temperature and at the fixed value of $\mu = 0.5\hbar\omega$.

We can not calculate the extremal points of the dispersion relation (14) in closed form; the results of [13] were found numerically. It is instructive to get an analytical expression for conductance, by considering the limit of small transverse component of the nuclear field. Moreover, let us consider the case when the chemical potential is such that the main time-dependent contribution to the conductance is due to the two local minima of the lowest subband; see Fig. 4. This condition is satisfied if the chemical potential μ is smaller than the local maximum point of the first subband and the temperature is sufficiently low. In this case, it is only necessary to know the time dependence of the two local minima in the dispersion relation.

Assuming that $B_{N,y}(t)$ is small, we can find approximate positions of the minima of the dispersion relation (14) by setting $B_y \equiv B_{N,y}(t) = 0$. The locations of the minima of (14) are

$$k_m = \pm\sqrt{\frac{\alpha^2 m^{*2}}{\hbar^4} - \frac{B_t^2\Gamma^2}{\alpha^2}}. \qquad (16)$$

We substitute (16) into (14) and expand the square root in (14) in Taylor series. Keeping only the first-order terms we find,

$$E = \frac{\hbar^2 k_m^2}{2m^*} + E_n^{tr} - \frac{\alpha^2 m^*}{\hbar^2} - \frac{k_m B_{N,y}(t)\Gamma\hbar^2}{\alpha m^*}. \qquad (17)$$

We have compared the time dependence of the minima found numerically vs. this approximation and found that for small $B_{N,y}(t)$ the approximation is quite reasonable.

We are now in position to find the conductance of the system. We are considering our system at low temperature and having chemical potential below the local maximum in the first subband. Therefore, to a good approximation the conductance can be expressed by (5) as,

$$G = \frac{e^2}{h}\left[f(E_1) + f(E_2)\right], \qquad (18)$$

where, E_1 and E_2 are the energies of the two local minima. If we note that (17) can be written as $E = E_0 \pm \Delta E$, where $E_0 = \hbar^2 k_m^2/(2m^*) + E_n^{tr} - \alpha^2 m^*/\hbar^2$, $\Delta E = |k_m| B_{N,y}(t)\Gamma\hbar^2/(\alpha m^*)$, and the $\pm$ correspond to the two extremal points, then by substitution of this into (18) we find,

$$G = \frac{2e^2}{h}\frac{1 + D\cosh(\beta(t))}{1 + D^2 + 2D\cosh(\beta(t))}. \qquad (19)$$

where $D = \exp\left[(E_0 - \mu)/(kT)\right]$, $\beta(t) = \Delta E/(kT)$.

The evolution of nuclear spin polarization at longer times results in smooth conductance variations on the time scale T_1. This effect is similar to the smooth conductance variations discussed in the next subsection.

B. High Magnetic Fields

Let us now consider the case when the applied magnetic field is large enough or generally the cases when the effects of the Rashba SO interaction can be neglected. By neglecting the third term in (13) and solving the Schrödinger equation via the adiabatic approximation discussed earlier, we find that the eigenvalues become

$$E_{n,\pm}(k) = \frac{\hbar^2 k^2}{2m^*} + E_n^{tr} \pm \Gamma B_t(t), \qquad (20)$$

where B_t is defined by

$$B_t^2(t) = \left[B + B_0 + (B_{N,x}(t=0) - B_0)e^{-t/T_1}\right]^2 + B_{N,y}^2(t=0)e^{-2t/T_2}, \qquad (21)$$

and B_0 is the equilibrium value of the effective nuclear field. Fig. 6 shows the time dependence of B_t for different values of θ, the angle in the (x, y) plane between the x axis and the initial direction of the nuclear spin polarization. Two different time scales are manifested in Fig. 6. The short time decrease of B_t is related to the transverse nuclear spin relaxation, governed by the T_2 time, while the long-time evolution is due to the longitudinal nuclear spin relaxation, which occurs on the time scale T_1. From the experimental point of view, the angle θ can be easily controlled by varying the NMR pulse width.

The subbands given by (20) have only one minimum point at $k = 0$, for each spin. The energy associated with this extremal point is simply $E_{n,\pm}(k = 0)$. Let us assume that the

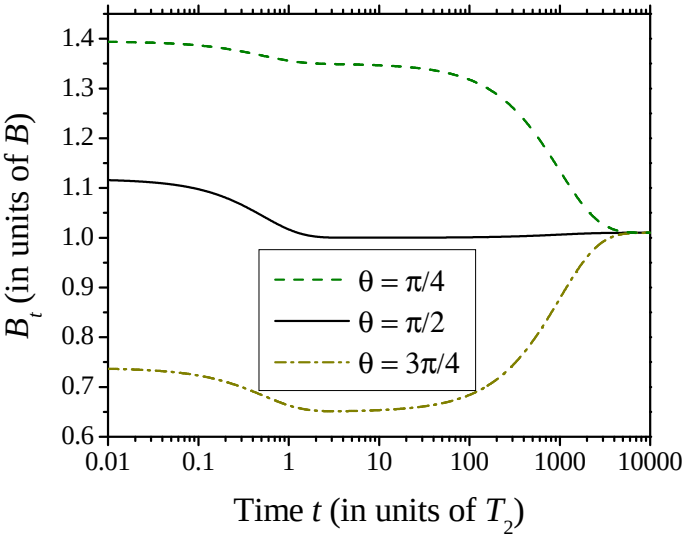

Fig. 6. Time dependence of B_t calculated for the following set of parameters: $B_0 = 0.01B$, $B_N(t=0) = 0.5B$. Different curves correspond to different initial directions of the nuclear spin polarization.

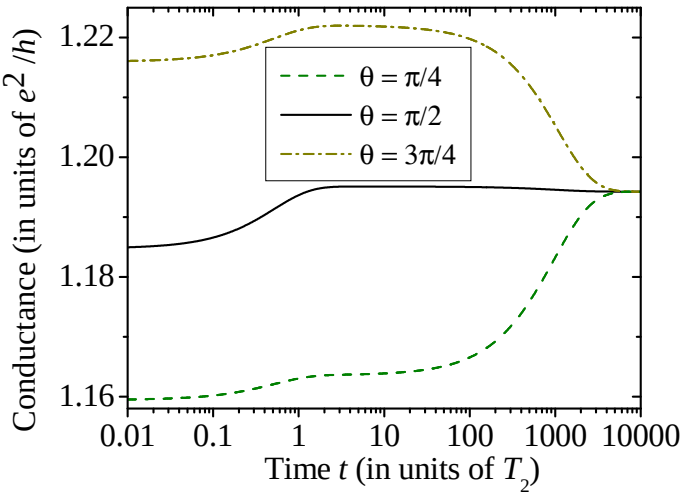

Fig. 7. Evolution of the conductance of the QW due to the relaxation of the spin-polarized nuclei. Here $(E_0^{tr} - \mu)/(kT) = 0.5$, $\Gamma B/(kT) = 1$, while the other parameters are the same as in Fig. 6.

distance between the transverse subbands is much larger than the temperature, $\hbar\omega \gg kT$, and that the chemical potential μ is close to the minimum point of a subband. The time-dependent contribution to the conductance, calculated by substituting of the extremal points into (5), is given by the same expression as (19), but with the following parameters: $\beta(t) = \Gamma B_t(t)/(kT)$ and $D = \exp\left((E_n^{tr} - \mu)/(kT)\right)$.

The time dependence of the conductance is defined by the time dependence of the magnitude of the total field B_t, initial magnitude of nuclear spin polarization B_N, its direction described by the angle θ, chemical potential, and temperature. The time-dependent conductance of the system is shown in Fig. (7) for selected values of parameters and different values of θ. For these parameter values, for short times the conductance G initially increases with time. However the long time behaviour is quite varying: conductance could be constant, increasing or decreasing, depending on the initial direction of the nuclear spin polarization. These interesting features open up a way to experimentally separate out the contribution of a selected nuclear isotope.

V. Conclusions

We have shown that via the time dependent conductance of a QW it may be possible to extract the transverse and longitudinal relaxation times of the spin-polarized nuclei in the channel of the QW. When the effects of the Rashba SO interaction are important, we find that the conductance exhibits damped oscillations. The oscillations have a frequency that is proportional to the gyromagnetic ratio of the nuclei and to the applied magnetic field. The time scale of the damping depends primarily on the time T_2. In the limit of a large applied magnetic field, when the effects of the Rashba SO interaction can be neglected, we have shown that the conductance oscillations disappear. Our results suggest that it may be possible to extract the T_1 and T_2 nuclear time scales from the experimental data. The possibility to control the initial nuclear spin polarization direction by an NMR pulse gives additional control variable for interpretations of the experimental data. The proposed technique can be useful for spin state readout in quantum computing applications [37]–[40].

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